18. Solve $z = px + qy + p^2q^2$.

19. If $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$. Evaluate $\iint_S \vec{F} \cdot \vec{n}\, dS$ where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

20. Obtain the Fourier series expansion of the function $f(x)$ with period 2π, defined by $f(x) = x$, $-\pi < x \leq \pi$.

NOVEMBER/DECEMBER 2024

FAEL22/CAEL22 — BASIC MATHEMATICS – II

Time : Three hours Maximum : 75 marks

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SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL the questions

1. Find the n^{th} derivative of a^x.

2. Define Jacobian.

3. Solve $(D^2 - 3D + 5)y = 0$.

4. What do you mean by particular solution?

5. Find the Partial Differential Equation of all planes having equal intercepts from the x and y axis.

6. Define singular integral.

7. Define a simple closed curve.

8. If C is the circle $x = a\cos\theta$, $y = a\sin\theta$, then find $\int_C x\, dy - y\, dx$.

9. Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$.

10. Find b_n in the expansion of x^2 as a Fourier series in $(-\pi, \pi)$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) Find the n^{th} derivative of $\frac{2x+1}{(2x-1)(2x+3)}$.

Or

(b) If $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

12. (a) Solve $(D^2 - 3D + 2)Y = e^{5x} + 2$.

Or

(b) Solve $(D^2 - 4D + 3)Y = x^3 e^{2x}$.

13. (a) Find the partial differential equation by eliminating the arbitrary constants a and b from $\log(az - 1) = x + ay + b$.

Or

(b) Solve $p^2 + q^2 = x + y$.

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14. (a) Show that the vector $3x^2y\vec{i} - 4xy^2\vec{j} + 2xyz\vec{k}$ is solenoidal.

Or

(b) Prove that $\nabla \times (\nabla r^n) = 0$.

15. (a) Find $\int x^2 e^{2x} dx$.

Or

(b) Prove that $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}$.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. If $y = (\sin^{-1} x)^2$ prove that

$$(1 - x)^2 y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0.$$

Hence find the value of y_n at $x = 0$.

17. Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{1}{1 + e^x}$.